Strong decays and Adler selfconsistency condition in two-dimensional QCD

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Abstract

Strong mesonic decays are studied in the framework of the 't Hooft model for the two-dimensional QCD in the axial gauge. Special attention is payed to the processes with pions in the final state involved and the low energy theorems have been checked including the "Adler zero". It is demonstrated explicitly that in the chiral limit any three-meson decay amplitude with at least one pion vertex vanishes identically for any values of the external momenta. Ward identities for the dressed vector and axial-vector currents vertices are identified and their relationship to the pionic vertex are discussed.

It has been recognized that strong decays play the crucial role in hadron spectroscopy issues. There are two reasons for this. First, as the distances involved in the light quark sector are large, the dominant mechanism of strong decay should be nonperturbative, so that the decay studies deliver knowledge on the picture of confinement. Another point is that strong hadronic decays are proved to be a rather important tool in searches for constituent glue. The characteristic decay selection rules are known which distinguish between *e.g.* $q\bar{q}$ and hybrid mesons [1]. These selection rules are derived on the basis of quark models, and the question remains unanswered of how reliable are the model predictions from the *QCD* motivated point of view.

In the constituent quark picture approach the confinement is modeled by a potential force, and the resulting spectrum describes the data reasonably well with notable exception, the pion. The chiral symmetry breaking (CSB) effects lie definitely beyond the scope of a simple constituent picture, and there is no hope to obtain Goldstone boson within any naive quark model. Similarly, decays with pions in the final state present problems for pair creation models. For example, in the standard $^3P_0$ model [2, 3, 4] a reasonable fit to experimental data is obtained only with the so-called "mock meson" prescription for the phase space [3], while for the correct relativistic phase space the rate for processes with pions in the final state is too low [4]. On the contrary, the microscopic model [5] of hadronic decays overestimates the decays with pions for higher quarkonia [6]. What is worse, the naive pair creation models are based on the constituent picture and do not respect soft pions theorems; in particular, there is no hope to obtain Adler selfconsistency condition [7] for the amplitude with pions.

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Unfortunately, the decays of immediate relevance for hadron spectroscopy are mostly the ones with pions in the final state [8].

Adler selfconsistency condition follows from the most general symmetry considerations, and has nothing to do with the particular mechanism of confinement. Nevertheless, the idea that confinement and CSB phenomena are interrelated seems to be rather meaningful. A model was suggested many years ago [9] which connects confinement and CSB (see also *e.g.* [10], where similar ideas were employed). Unfortunately, this model main ingredient, three-dimensional oscillator confining force, is not motivated by QCD, and among other drawbacks of such an approach is lack of gauge and Lorentz invariance. An important step was made in [11], where it was shown that the same gauge and Lorentz invariant nonperturbative gluonic correlators which produce area law are responsible for the formation of chiral quark condensate. Thus the quark model motivated by such a picture should be free of above-mentioned drawbacks and should be able to reproduce, *inter alia*, all pion properties. Obviously this quark model is going to be rather unusual, and it is instructive to study strong decays in some toy exactly solvable theory with confinement and CSB.

An example exists of a theory where both confinement and CSB are due to the same force, and it is two-dimensional quantum chromodynamics (QCD_2) in the large N_C limit which was first considered many years ago [12] and still remains popular in studies of various aspects of strong interactions. The large N_C limit is important in establishing the chiral properties of the theory. Indeed, it is well known that the Coleman theorem [13] prohibits CSB for any finite number of colours in a two-dimensional theory. Still there is no contradiction with the Coleman theorem [13] if one considers the weak coupling regime of the theory where $m_q \gg g \sim 1/\sqrt{N_C}$, *i.e.* the limit of infinite number of colours is taken first (see *e.g.* [14] for detailed discussion of this issue as well as of another phase of the theory which corresponds to the strong coupling regime $m_q \ll g$ and where the Coleman theorem works at full scale).

Most of the studies in QCD_2 were performed in the light-cone gauge, which considerably simplifies the spectrum calculations but yields perturbative vacuum. There exists an alternative approach based on the Coulomb gauge $A_1 = 0$ [15]. It appears that the vacuum is nontrivial in the Coulomb gauge version, and nonzero quark condensate exists for massless quarks [16]. The latter feature is confirmed by sum rules approach [14] in the light-cone gauge.

In the present letter we perform studies of strong hadronic decays in the QCD_2 employing the Coulomb gauge. This choice enables treating pions on the same footing as other mesons. The latter is in contrast to light-cone gauge choice, where the pion wave function is very singular, and one is forced to use sophisticated nonperturbative methods, like sum rules and operator product expansion, to arrive at reliable results in the pion physics [14].

Let us review the essentials of QCD_2 in the Coulomb gauge (see [15] for the details). Our convention for γ matrices is $\gamma_0 = \sigma_3$, $\gamma_1 = i\sigma_2$, $\gamma_5 = \gamma_0\gamma_1$. The large N_C limit implies that $g^2 N_C$ remains finite. The gluonic propagator in the Coulomb gauge $A_1 = 0$ takes the form $D_{00}(k_0, k) = -1/k^2$, and the infrared singularity is regularized by the principal value prescription yielding the linear confinement. There are no transverse gluons in two dimensions, and in the large N_C limit only planar graphs survive, so the only nontrivial one-particle quantity is the quark Green's function

$$S(p_0, p) = \frac{1}{\hat{p} - m - \Sigma(p)}, \tag{1}$$

which can be found from the Schwinger–Dyson equation with the result

$$\Sigma(p) = [E(p)\cos\theta(p) - m] + \gamma_1[E(p)\sin\theta(p) - p], \tag{2}$$

$$p\cos\theta(p) - m\sin\theta(p) = \frac{\gamma}{2}\!\!\int \frac{dk}{(p-k)^2}\sin[\theta(p) - \theta(k)], \tag{3}$$

$$E(p) = m\cos\theta(p) + p\sin\theta(p) + \frac{\gamma}{2}\!\!\int \frac{dk}{(p-k)^2}\cos[\theta(p) - \theta(k)], \tag{4}$$

where $\gamma = \frac{g^2 N_C}{4\pi}$. Parameter $\theta(p)$ has the meaning of the Bogoliubov–Valatin angle describing the rotation from bare to dressed quarks. The obvious properties of the solutions to the equations (3), (4) are $\theta(-p) = -\theta(p)$, $E(-p) = E(p)$, and $\theta(p) \underset{p\to\infty}{\to} \pi/2$.

The spectrum of the theory is defined from the homogeneous Bethe–Salpeter equation for the matrix wave function

$$\Phi(p, P) = T(p)\left(\frac{1+\gamma_0}{2}\gamma_5\varphi_+(p, P) + \frac{1-\gamma_0}{2}\gamma_5\varphi_-(p, P)\right)T^+(P - p) \tag{5}$$

with $T(p) = exp(-\frac{1}{2}\theta(p)\gamma_1)$. The functions $\varphi_\pm(p, P)$ are solutions to the system of equations

$$\begin{cases} [E(p) + E(P - p) - P_0]\varphi_+(p, P) \\ \qquad = \gamma\!\!\int \frac{dk}{(p-k)^2}\left[C(p, k, P)\varphi_+(k, P) - S(p, k, P)\varphi_-(k, P)\right] \\ \\ [E(p) + E(P - p) + P_0]\varphi_-(p, P) \\ \qquad = \gamma\!\!\int \frac{dk}{(p-k)^2}\left[C(p, k, P)\varphi_-(k, P) - S(p, k, P)\varphi_+(k, P)\right], \end{cases} \tag{6}$$

where

$$C(p, k, P) = \cos\frac{\theta(p) - \theta(k)}{2}\cos\frac{\theta(P - p) - \theta(P - k)}{2},$$

$$S(p, k, P) = \sin\frac{\theta(p) - \theta(k)}{2}\sin\frac{\theta(P - p) - \theta(P - k)}{2}, \tag{7}$$

with P and p being the total meson momentum and the momentum of the quark respectively.

The system (6) describes a meson moving forward in time as a superposition of the quark-antiquark pair moving forward in time with the amplitude φ_+ and backward in time with the amplitude φ_-. Such a particle-hole interpretation is supported by the Hamiltonian approach developed in [17], where the operator $m_n^+(P)$ creating the n-th meson with total momentum P was constructed as a linear combination of operators creating and annihilating the $q\bar{q}$ pair.

A very interesting point concerning Bars–Green system of equations (6) is that if it is treated as a matrix integral equation, then its kernel is not Hermitian [17]. The eigenvalues P_0^n are real, but the orthonormality and completeness conditions take the form (see [17] for details)

$$\int \frac{dp}{2\pi}\left(\varphi_+^n(p, P)\varphi_+^m(p, P) - \varphi_-^n(p, P)\varphi_-^m(p, P)\right) = \delta_{nm}$$

$$\int \frac{dp}{2\pi}\left(\varphi_+^n(p, P)\varphi_-^m(p, P) - \varphi_-^n(p, P)\varphi_+^m(p, P)\right) = 0 \tag{8}$$

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$$\sum_{n=0}^{\infty} \left(\varphi_+^n(p, P)\varphi_+^n(k, P) - \varphi_-^n(p, P)\varphi_-^n(k, P) \right) = 2\pi\delta\left(p - k\right)$$

$$\sum_{n=0}^{\infty} \left(\varphi_+^n(p, P)\varphi_-^n(k, P) - \varphi_-^n(p, P)\varphi_+^n(k, P) \right) = 0. \tag{9}$$

Solutions of the system (6) come in pairs: for each eigenvalue P_0^n with eigenfunction $(\varphi_+^n, \varphi_-^n)$ there exists another eigenvalue $-P_0^n$ with eigenfunction $(\varphi_-^n, \varphi_+^n)$. With this symmetry only positive eigenvalues enter the forms (8), (9).

The system (6) is solved numerically in [18], and it is shown that the φ_- component is small i) if the quark mass is large and ii) for higher excited states. In both cases quark potential model with local linear confinement serves as a good approximation. Besides that the φ_- component dies out with the increase of the total mesonic momentum P: in the infinite momentum frame ($P \to \infty$) it is zero and the equation for the φ_+ reproduces the 't Hooft equation [12] after an appropriate rescaling, as it was shown in [15].

The chiral condensate is calculated straightforwardly from equation (1):

$$\langle vac|q\bar{q}|vac \rangle = -\frac{N_C}{2\pi} \int_{-\infty}^{+\infty} dk \cos \theta(k). \tag{10}$$

It is shown in [16] that the gap equation (2) has nontrivial solution in the chiral limit $m = 0$, and the chiral condensate (10) does not vanish with this solution. It means that the Goldstone mode exists in the spectrum, and its explicit form was found in [17]:

$$\varphi_{\pm}^{\pi}(p, P) = \sqrt{\frac{\pi}{2P}} \left(\cos \frac{\theta(P - p) - \theta(p)}{2} \pm \sin \frac{\theta(P - p) + \theta(p)}{2} \right) \tag{11}$$

for $P > 0$ and $P_0 = \sqrt{P^2}$ ($\varphi_-^{\pi} \leftrightarrow \varphi_+^{\pi}$ for $P < 0$). For $P = 0$ $\varphi_+^{\pi}(p, 0) = \varphi_-^{\pi}(p, 0)$, pion spends half of time in backward motion of the pair, and such a function has zero norm, as it should be for the massless particle at rest. In the opposite limiting case $P \to \infty$ the backward motion part dies out, and

$$\varphi_+^{\pi}(p, P) = \sqrt{\frac{2\pi}{P}}, \quad 0 \le p \le P, \tag{12}$$

coinciding with the Goldstone mode of the 't Hooft equation. Form (12) is the main reason to consider the Coulomb gauge version of the theory, as all nontrivial content of the wave function (12) is concentrated in the boundary regions $x \to 0$ and $x \to 1$ ($x = p/P$). The same is true of course for the QCD_2 quantized at the light-cone [19]. Quantities like chiral condensate do not come out trivially with such a singular wave function [14].

Now we are in the position to calculate the strong decay amplitude (Fig.1), which is $O(1/\sqrt{N_C})$. The answer reads

$$M(A \to B + C) =$$

$$-\frac{i\gamma^3}{\sqrt{N_C}} \int \frac{d^2k}{(2\pi)^2} Sp(\Gamma_A(k + P_B, P_A)S(k - P_C)\bar{\Gamma}_C(k, P_C)S(k)\bar{\Gamma}_B(k + P_B, P_B)S(k + P_B)) \tag{13}$$

$$+(B \leftrightarrow C),$$
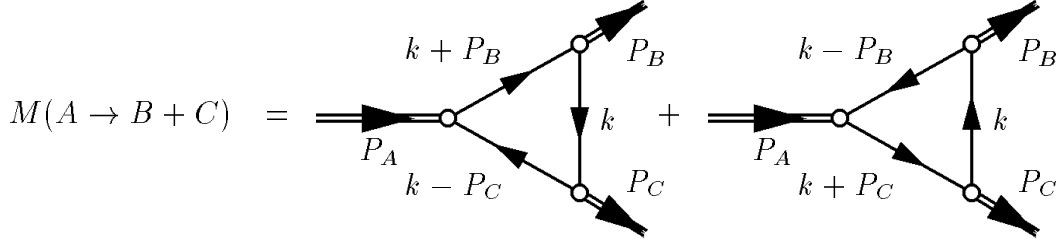
$$M(A \to B + C) \quad = \quad$$

Figure 1: Graphical representation of the decay amplitude (13).

where the meson-quark-antiquark vertex Γ_M can be expressed in terms of the Bethe–Salpeter wave function (5)

$$\Gamma_M(p, P) = \int \frac{dk}{2\pi} \gamma_0 \frac{\Phi_M(k, P)}{(p - k)^2} \gamma_0, \qquad (14)$$

and $\bar{\Gamma}_M(p, P) = \gamma_0 \Gamma_M^+(p, P)\gamma_0$. As the vertex (14) does not depend on p_0, the integration along the loop in (13) is trivial, so one arrives at the following final expression for the decay amplitude in the initial meson rest frame $P_A = 0$, $P_B = -P_C = p$:

$$M(A \to B + C) = \qquad (15)$$

$$\frac{\gamma}{\sqrt{N_C}} \int \frac{dkdq}{(q - k)^2} \{ - \varphi_-^A(k + p, 0)\varphi_-^B(k + p, 0)[c(-p, q, k)\varphi_+^C(q, -p) + s(-p, q, k)\varphi_{-C}]$$

$$-\varphi_+^A(k + p, 0)\varphi_+^C(k, -p)[c(p, q + p, k + p)\varphi_+^B(q + p, p) + s(p, q + p, k + p)\varphi_-^B(q + p, p)]$$

$$-\varphi_+^C(k, -p)\varphi_-^B(k + p, p)[s(0, q + p, k + p)\varphi_+^A(q + p, 0) + c(0, q + p, k + p)\varphi_-^A(q + p, 0)]$$

$$+\varphi_-^C(k, -p)\varphi_+^B(k + p, p)[c(0, q + p, k + p)\varphi_+^A(q + p, 0) + s(0, q + p, k + p)\varphi_-^A(q + p, 0)]$$

$$+\varphi_-^A(k + p, 0)\varphi_-^C(k, -p)[s(p, q + p, k + p)\varphi_+^B(q + p, p) + c(p, q + p, k + p)\varphi_-^B(q + p, p)]$$

$$+\varphi_+^A(k + p, 0)\varphi_+^B(k + p, p)[s(-p, q, k)\varphi_+^C(q, -p) + c(-p, q, k)\varphi_-^C(q, -p)] \}$$

$$+(B \leftrightarrow C, p \leftrightarrow -p),$$

where

$$c(p, q, k) = \cos \frac{\theta(k) - \theta(q)}{2} \sin \frac{\theta(p - k) - \theta(p - q)}{2},$$

$$s(p, q, k) = \sin \frac{\theta(k) - \theta(q)}{2} \cos \frac{\theta(p - k) - \theta(p - q)}{2}.$$

Amplitude (15) could be easily obtained from the Hamiltonian approach developed in [17], though further calculations appear more transparent using the matrix wave function (5) and the matrix form of the bound state equation (6). Nevertheless, equivalence of both approaches can be established at any intermediate step.

If one neglects the backward motion contributions in the amplitude (15) and inserts the nonrelativistic values of the angle θ ($\cos \theta(k) = 1, \sin \theta(k) = k/m$) then it reproduces the standard quark model decay amplitude due to OGE Coulomb interaction [5] adapted to the two-dimensional case. It is clear, however, that substitution of the nonrelativistic angle is not justified for kinematical reasons.

The six-term form (15) was anticipated in [20], where the role of backward motion in decay was discussed for the first time. It was argued there that a simple procedure could
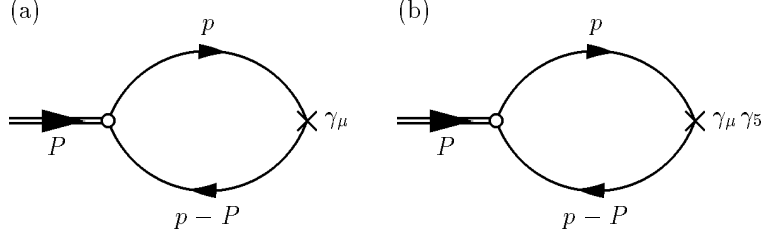
Figure 2: The meson-vector (a) and meson-axial-vector (b) current couplings.

be used as a crude approximation. Namely, one neglects the backward motion pieces for all mesons but pions. Then a simple graph counting would give the net result: amplitudes with a pion in the final state are multiplied by an overall factor 2 over the naive quark model result, and those with two pions are multiplied by 3. Obviously this prescription of constructive interference does not solve the Adler selfconsistency problem.

It appears that in the two-dimensional amplitude (15) the interference between the six terms is destructive: **the total amplitude vanishes in the chiral limit, if at least one of the final state mesons is the pion**. Of course, one can perform explicit calculations inserting the pion w.f. (11) and making use of the equations (3), (4) and (6). There exists, however, a simple and elegant expression for the pion-quark-antiquark vertex in the limit $m = 0$:

$$\Gamma_\pi(p, P) = S^{-1}(p)(1 + \gamma_5) - (1 - \gamma_5)S^{-1}(p - P). \tag{16}$$

Inserting it into equation (13) trivially yields $M(A \to \pi + C) = 0$ in the chiral limit.

Expression (16) for the pion vertex follows from rather general considerations and is connected with the vector and axial-vector currents conservation.

The meson-vector current coupling $V_\mu^M(P) = \langle vac|\bar{q}\gamma_\mu q|M, P\rangle$ can be calculated as (Fig.2a)

$$V_\mu^M = i\gamma\sqrt{N_C} \int \frac{d^2p}{(2\pi)^2} Sp(S(p - P)\gamma_\mu S(p)\Gamma_M(p, P)) = \sqrt{N_C} \int \frac{dp}{2\pi} Sp(\gamma_\mu \Phi_M(p, P)). \tag{17}$$

Substituting expression (5), one finds

$$V_0^M = -\sqrt{N_C} \int \frac{dp}{2\pi} [\varphi_+^M(p, P) + \varphi_-^M(p, P)] \sin\frac{\theta(P - p) + \theta(p)}{2},$$

$$V^M = -\sqrt{N_C} \int \frac{dp}{2\pi} [\varphi_+^M(p, P) - \varphi_-^M(p, P)] \cos\frac{\theta(P - p) - \theta(p)}{2}. \tag{18}$$

To demonstrate the vector current conservation we write down the equation for the matrix wave function (5) as

$$P_0^M \Phi_M(p, P) = (\gamma_5 p + \gamma_0 m)\Phi_M(p, P) - \Phi_M(p, P)(\gamma_5(P - p) + \gamma_0 m)$$

$$+ \gamma \int \frac{dk}{(p - k)^2} \{\Lambda_+(k)\Phi_M(p, P)\Lambda_-(P - k) - \Lambda_+(p)\Phi_M(k, P)\Lambda_-(P - p) \tag{19}$$

$$-\Lambda_-(k)\Phi_M(p, P)\Lambda_+(P - k) + \Lambda_-(p)\Phi_M(k, P)\Lambda_+(P - p)\},$$

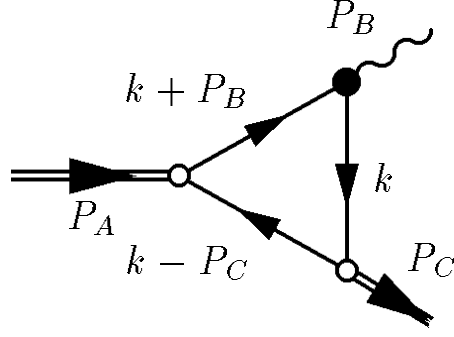where projectors are defined as $\Lambda_\pm(p) = T(p)\frac{1 \pm \gamma_0}{2}T^+(p)$.

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Figure 3: Graphical representation of the mesonic form factor.

Then, integrating this equation over p, multiplying by γ_0 and taking trace one gets

$$P_0^M V_0^M - PV^M = 0, \tag{20}$$

i.e. the vector current is conserved.

Similarly, for the axial-vector current matrix element $A_\mu^M(P) = \langle vac|\bar{q}\gamma_\mu\gamma_5 q|M, P\rangle$ one can find (Fig.2b)

$$P_0^M A_0^M - PA^M = -2m\sqrt{N_C} \int \frac{dk}{2\pi} Sp(\Phi), \tag{21}$$

that yields axial-vector current conservation for $m = 0$. Slightly relaxing the chiral limit one gets the pionic decay constant $f_\pi = \sqrt{\frac{N_C}{\pi}}$ and the Gell-Mann–Oaks–Renner relation [21]

$$f_\pi^2 M_\pi^2 = -2m\langle\bar{q}q\rangle \tag{22}$$

(see also [14, 17]). It is instructive to mention that in the chiral limit vector current has nonzero coupling only to the pion: the vector current matrix elements (18) contain the meson wave functions integrated with the pionic one (11), so the corresponding decay constant vanishes due to orthogonality conditions (8). (We remind here that the pion does couple to the vector current in 1+1!). As the axial-vector current in 1+1 is dual to the vector current, the axial decay constants f_M for excited mesons vanish in the chiral limit too.

Calculations of mesonic form factors (Fig.3) are more involved, and require knowledge of the full quark-antiquark scattering amplitude $\Gamma_{ik,lm}(p_\mu, k_\mu, P_\mu)$,

$$\Gamma_{ik,lm}(p_\mu, k_\mu, P_\mu) = \frac{2\pi i\gamma}{(p-k)^2}(\gamma_0)_{il}(\gamma_0)_{km} - i\gamma^2\sum_M \frac{1}{P_0 - P_0^M}(\Gamma_M(p, P))_{im}(\bar{\Gamma}_M(k, P)_{kl},$$

$$+ i\gamma^2\sum_M \frac{1}{P_0 + P_0^M}(\gamma_0\bar{\Gamma}_M(P-p)\gamma_0)_{im}(\gamma_0\Gamma_M(P-k, P)\gamma_0)_{kl}. \tag{23}$$

which is the solution of the inhomogeneous Bethe–Salpeter equation.

The dressed vector current-quark-antiquark vertex v_μ (Fig.4) is then given by

$$v_\mu(p, P_\mu) = i\gamma_\mu + i\gamma\sum_M \frac{1}{P_0 - P_0^M} \int \frac{dq}{2\pi} Sp[\gamma_\mu\Phi_M(q, P)]\bar{\Gamma}_M(p, P)$$

$$- i\gamma\sum_M \frac{1}{P_0 + P_0^M} \int \frac{dq}{2\pi} Sp[\gamma_\mu\Phi_M^+(P-q, P)]\gamma_0\Gamma_M(P-p, P)\gamma_0, \tag{24}$$
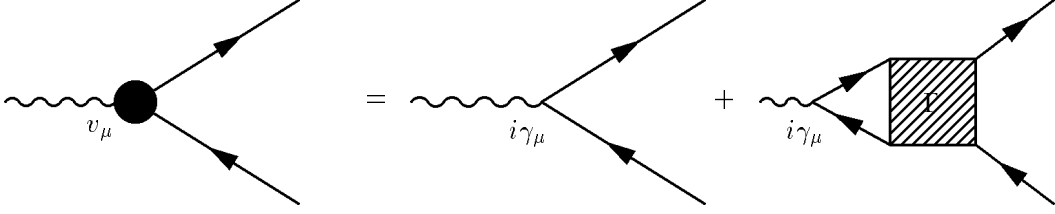
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Figure 4: Graphical representation of the dressed vector current-quark-antiquark vertex.

where P is the total momentum of the quark-antiquark pair, and p is the momentum of the quark. One easily recognizes the vector current couplings (17) entering (24), so that in the chiral limit only pion contributes into v_μ, displaying a sort of "vector dominance". The same takes place with the axial-vector current-quark-antiquark vertex.

Using the equations (1), (2) and (5) together with the completeness condition (9) allows to recast the vector current divergence into the Ward identity form (similar expression in the light–cone gauge was derived in [22]).

$$- iP_\mu v_\mu(p, P) = S^{-1}(p) - S^{-1}(p - P), \tag{25}$$

which makes vector current conservation manifest for the form factors:

$$Q_\mu \langle M, P | v_\mu | M', P' \rangle = 0, \quad Q_\mu = P_\mu - P'_\mu. \tag{26}$$

Similarly, the axial-vector current divergence in the chiral limit is

$$- iP_\mu a_\mu(p, P) = S^{-1}(p)\gamma_5 + \gamma_5 S^{-1}(p - P), \tag{27}$$

and

$$Q_\mu \langle M, P | a_\mu | M', P' \rangle = 0, \quad Q_\mu = P_\mu - P'_\mu. \tag{28}$$

The pion-quark-antiquark vertex (16) is nothing but a linear combination of the vector and the axial-vector currents divergences (25) and (27)

$$\Gamma_\pi(p, P) = -iP_\mu v_\mu(p, P) - iP_\mu a_\mu(p, P). \tag{29}$$

Relation (29) plays the role of PCAC hypothesis in 1+1: as pion couples both to vector and axial-vector currents, the form (29) is inevitable in 1+1 then.

In conclusion, let us briefly recall the results reported in the present letter. In the previous publication [17] we developed a Hamiltonian approach to the two-dimensional QCD in the axial gauge and demonstrated how the Hamiltonian of the model could be diagonalized in the mesonic sector. One of the key features of the above Hamiltonian is the distorted norm (8) plugged into it which allows description of the massless $q\bar{q}$ states, chiral pions, on equal footing with the rest of higher massive excitations. Chiral properties of the model were reproduced and discussed. In the meantime a crucial role of the backward in time motion of the $q\bar{q}$ pair inside meson, extremely important for the pion, was anticipated but not discussed in detail. In the present letter we return to this issue and investigate the role played by pions in the strong hadronic decays and in providing the low energy theorems like "Adler zero" selfconsistence condition. We use an effective diagrammatic techniques involving dressed meson-quark-antiquark vertex, quark and mesonic propagators and the full four-quark scattering amplitude. Any hadronic process can be formulated diagrammatically

using the above ingredients. The three-meson amplitude is studied in detail in the suggested approach and a strict result is obtained stating that the interference between different parts of this amplitude which are due to φ_+ and φ_- components of the pion is completely destructive, so that the total amplitude vanishes for any pion momentum, not only in the limit $p_\pi \to 0$. The latter result seems quite natural if one takes into account that the pionic decay constant f_π is dimensionless in 1+1 and does not define any scale for soft pions then.

Developing similar approach for the four-dimensional QCD, which could shed light on many puzzles of hadronic spectroscopy, is still an open challenge. In the model [9] the interaction is described by the time component of a vector force, yielding the gap equation similar to (3), (4) and the Bethe–Salpeter equation similar to (19). The existence of a chiral-noninvariant solution of the gap equation implies the existence of a Goldstone boson. Axial-vector current is conserved in the chiral limit of the model, and all the relations of current algebra are satisfied. However, the model is not covariant, that prevents from proceeding further along the lines presented here. We point out once more that the approach suggested in [11] is rather promising: the gauge invariant confining interaction employed in [11] is Lorentz covariant by construction, respects the area law and is responsible for the chiral condensate formation. We expect that the quark model derived within such a formalism could be able to reproduce all the nice features described above and, being fully covariant, could allow to calculate hadronic observables.

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